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EFFECTIVE TARGET MASS IN PION-PROTON INTERACTIONS

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EFFECTIVE TARGET MASS IN PION-PROTON INTERACTIONS

CHAPTER I

Introduction

The fundamental problem of nuclear physics is to determine the nature of the forces between nucleons. This problem is made difficult by the fact that the force is of very short range and cannot be observed in detail in the same manner that gravitational and electrostatic forces can be observed.

Several potentials have been suggested to account for the nuclear force. The most physical of these has been the one of Yukawa who suggested a potential of the form

$$V(r) = g^2 \frac{e^{-r/r_o}}{r}$$

in which r_o is the range of the force field and g is the coupling constant analogous to charge in electrostatic fields. If one assumes the energy of the nuclear force field to be quantized in the same manner as energy in the electromagnetic field, then there must be quanta associated with the nuclear force field analogous to photons in the electromagnetic field. The mass of these quanta can be related to the range of the force by the relation for Compton wavelength, $r_o = \frac{\hbar}{mc}$. The quantity r_o has been determined experimentally to be of the order of

10^{-13}cm. The mass, using this value for r_o, is calculated to be very close to the mass of the pion, 139.6 MeV/c^2, and it is suggested that pions are indeed the quanta for nuclear force fields. The general idea of this force field is that these pions are continuously being virtually emitted and absorbed by nucleons, thereby creating the field around the nucleons.

This idea has led to the hypothesis that a nucleon consists of a dense core surrounded by a virtual pion cloud. If this is the case, interactions with nucleons might appear to be interactions with pions in somewhat the same way that interactions with atoms are often seen to be interactions with electrons.

One might test the validity of having virtual pions as targets in high energy collisions by examining pion-nucleon interactions in which the nucleon recoils with low energy. In this situation the nucleon is treated as a spectator to an interaction between the pion and some effective target within the nucleon. The effective target mass in an interaction of this sort can be determined experimentally by measuring the momenta and angles of the secondary particles from the interactions, excluding the recoil nucleons.

Before the interaction we have, in the laboratory system, a pion with momentum $\vec{P}_o$ approaching a target which is assumed to be at rest.

By definition of the center of mass system[1],

$$\vec{P}'_o + \vec{P}'_{TGT} = 0 \ . \tag{1}$$

Laboratory and center of mass quantities can be related by Lorentz transformation[2],

$$P'_o = \gamma_c (P_o - \beta_c E_o)$$

$$P'_{TGT} = \gamma_c (P_{TGT} - \beta_c E_{TGT}) = -\gamma_c \beta_c M_{TGT}$$

where $\beta_c = v/c$, v is the laboratory velocity of the center of mass, E is energy and

$$\gamma_c = \frac{1}{\sqrt{1-\beta_c^{\,2}}} \ .$$

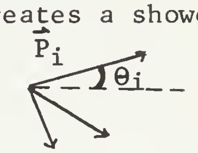
Then, from equation (1),

$$\gamma_c (P_o - \beta_c E_o) = \gamma_c \beta_c M_{TGT}$$

and,

$$M_{TGT} = \frac{P_o - \beta_c E_o}{\beta_c} \ . \tag{2}$$

The interaction creates a shower of secondary particles.



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[1]Primed quantities refer to the center of mass system; unprimed quantities to the laboratory system.

[2]In this thesis, a system of units is used such that c = 1.

Here P_i and θ_i are the laboratory momentum and space angle of the $i^{\underline{th}}$ particle. By Lorentz transformation to the center of mass system,

$$P'_i \cos \theta'_i = \gamma_c (P_i \cos \theta_i - \beta_c E_i) \quad . \tag{3}$$

Also, if one sums over all final state particles,

$$\sum \vec{P}'_i = 0 \quad .$$

Hence,

$$\sum P'_i \cos \theta'_i = 0$$

and, by equation (3),

$$\sum \gamma_c (P_i \cos \theta_i - \beta_c E_i) = 0 \quad .$$

Therefore, β_c is given by

$$\beta_c = \frac{\sum P_i \cos \theta_i}{\sum E_i} \tag{4}$$

and M_{TGT} can be determined from equation (2). This method of determining M_{TGT} will be referred to as Method I.

If one uses only one event to determine β_c in this manner, it is necessary to know the mass, the momentum, and the direction of each particle in the event. This is, in general, not possible since neutral particles which leave no tracks in the emulsion are almost always emitted. However, if one uses a large number of tracks from many events and assumes that neutral pions have the same angular and energy distributions as charged pions, then it is necessary only to know the masses, momenta, and directions of a representative set of tracks since

β_c is a ratio of sums and does not depend on the number of tracks involved.

Since one cannot, in general, measure the momentum of every particle and, perhaps, may not even be able to measure the momenta of a representative sample of particles, an alternative formulation would be of use. Investigators(1),(2),(3),(4) have found evidence that the average transverse momentum for shower particles in high energy interactions is very nearly independent of space angle. If this is true, then transverse momentum for each track in a group can be considered a constant equal to the average transverse momentum, and an alternative formulation for β_c in terms of angles and masses is obtained.

Thus, if one substitutes

$$P_T = P_i \sin \theta_i \quad \text{in equation (1),}$$

then

$$\beta_c = \frac{\sum \cot \theta_i}{\sum \left(\csc^2 \theta_i + \frac{m_i^2}{P_T^2}\right)^{\frac{1}{2}}} \; .$$

Determination of M_{TGT} using this β_c will be referred to as Method II.

The two methods described above for determining β_c can be combined to give a single value of β_c for a group of particles for which the measurement of momentum is possible for only a percentage of the group. In this method one can use the average transverse momentum of the particles with measured momenta to estimate the momenta of the remaining particles. Rather than use the average transverse momentum calculated for all particles, it is better to use an average transverse

momentum appropriate to a certain range of space angles to estimate unmeasured momenta in this angular range. Unmeasured momenta are determined by

$$P_i = \frac{\overline{P}_T}{\sin \theta_i}$$

where $\overline{P}_T$ is the average transverse momentum in the angular region containing θ_i.

Using this method all charged secondary particles have a value for momentum, and equation (4) can be used to determine β_c. Determination of M_{TGT} using this β_c will be referred to as Method III.

The accuracy of these three methods of determining effective target mass can be examined by calculating the total target mass using each method. If the recoil nucleons are included in the calculations of β_c, the resultant target mass should be that of a nucleon.

β_c then becomes

$$\beta_c = \frac{\sum P_i \cos \theta_i + \sum P_n \cos \theta_n}{\sum E_i + \sum E_n} \tag{5}$$

for Methods I and III, and

$$\beta_c = \frac{P_T \sum \cos \theta_i + \sum P_n \cos \theta_n}{P_T \sum (\csc^2 \theta_i + \frac{M_i^2}{P_T^2})^{\frac{1}{2}} + \sum E_n} \tag{6}$$

for Method II. The subscript n refers to nucleons.

It will be noted that none of these methods need use the entire set of secondary particles, but only a representative set. It is necessary, in general, to weight the sums over the secondary particles

of the representative set to achieve a proper proportion of pions to nucleons. Call this weighting factor f. Then

$$\beta_c = \frac{f\sum P_i \cos\theta_i + \sum P_n \cos\theta_n}{f\sum E_i + \sum E_n}$$

for Methods I and III, and

$$\beta_c = \frac{f\, P_T \sum \cot\theta_i + \sum P_n \cos\theta_n}{f\, P_T \sum \left(\csc^2\theta_i + \frac{m_i^2}{P_T^2}\right)^{\frac{1}{2}} + \sum E_n}$$

for Method II.

Several investigators(5),(6),(7),(8) have used formulations similar to these to obtain values for the effective target mass which are very close to the mass of the pion. This has given strength to the argument that an interaction between a pion and a nucleon is sometimes an interaction between a pion and the pion cloud surrounding the nucleon.

Some theoreticians(9) argue that the effective target mass cannot be related to the pion cloud and the fact that the effective target mass is of the order of the pion mass is a kinematical accident. If the reported value for this mass is truly a kinematical accident, a determination of the effective target mass using separate groups of pion-nucleon interactions in which the kinematics differ distinctly should yield distinctly different results. The purpose of this thesis is to examine interactions between pions and protons with these ideas in mind.

CHAPTER II

Experimental Procedure

Equipment

The experiment consisted of examination and analysis of pion-proton interactions in Ilford K-5 emulsions which had been exposed to a 16.2 ± 0.64 BeV/c negative particle beam at CERN. The beam consisted of greater than 90 per cent negative pions, with a small admixture of muons, kaons, and antiprotons. The emulsion stack, consisting of 56 plates with dimensions of 7.5 cm x 15.0 cm x 600 microns, was one-third of the Berkeley Я stack.

Scanning for interactions was accomplished by the along-the-track method using a Spencer binocular microscope mounted on a special stage constructed in the University of Oklahoma physics department machine shop. A 15 X Compens eyepiece was used in conjunction with a 55 X Koristka objective. Scanning speed was controlled at 20 cm/hr by an electrically driven stage. Minimum ionizing tracks in the beam direction were picked up about 4.0 mm from where they entered an emulsion plate and followed until they either interacted or left the plate.

A Leitz-Wetzler Ortholux binocular microscope head with Leitz focusing mechanism was used for detailed examination of events. This head was mounted on a traveling stage also constructed in the physics department machine shop. The stage was painstakingly refined to

eliminate noise to allow the microscope to be used for multiple Coulomb scattering measurements. Deviation of the stage motion from a straight line was sufficiently small to allow reasonably accurate measurement of the momenta of 16.2 BeV pion tracks in the emulsion using cell lenght of about 1500 microns. Ames micrometer dials with an accuracy of about 0.1 microns were attached to the stage for lateral measurements in the plane of the emulsion. Eyepiece and objective magnifications used varied from 10 x 10 to 12.5 x 100. Scattering measurements were made with a Leitz 12.5 X monocular micrometer eyepiece in conjunction with a Lietz 100 X objective. Accuracy of the eyepiece at this magnification is estimated to be better than .003 microns.

Selection of Events

A variety of interactions, including pion-proton, pion-neutron, pion-electron, and pion-nucleus, may occur in the emulsion. It is the pion-proton interaction that is of interest in this paper and one must use care in the selection of events which fit this category. It is not possible to be absolutely certain that a given event is a pion-proton interaction. However, it is possible to eliminate an event from consideration if it does not conform to well known conservation laws and kinematical principles.

Charge conservation is one of the more obvious characteristics to satisfy. However, even in this one may not be certain in the selection. The event must have an even number of tracks if all secondaries are singly charged since the total charge before the collision is zero, and the charges of an odd number of singly-charged

particles cannot add to zero. This does not insure compliance since negatively charged particles cannot be distinguished from positively charged particles in the emulsion, and the charges of an even number of tracks may add to something other than zero. Events which have a dark blob at the point of interaction can be classified as pion-nucleus collisions and therefore eliminated from consideration. Events with low-energy electron tracks, which are usually easily distinguishable, can also be eliminated since these tracks are those of Auger electrons which are associated with interactions with heavy nuclei. Conservation of baryons demands that, unless $p\,\bar{p}$ production occurs, no more than one proton be included in the secondary particles. This effectively eliminates all events with more than one identifiable proton.

Events with a proton track at a angle greater than 90° from the direction of motion of the incident pion can be eliminated kinematically. Using notation defined above one obtains,

$$P'_p \cos \theta'_p = \frac{P_p \cos \theta_p}{\gamma_c} - \beta_c E'_p \quad ,$$

where the subscript p refers to the proton. Now, if one substitutes

$$P_p \cos \theta_p = P_p \sin \theta \cot \theta_p = P'_p \sin \theta'_p \cot \theta_p \quad ,$$

then

$$P'_p \cos \theta'_p = \frac{P'_p \sin \theta'_p \cot \theta_p}{\gamma_c} - \beta_c E'_p$$

and

$$\frac{\cot \theta_p}{\gamma_c} = \frac{\beta_c}{\beta'_p} \csc \theta'_p + \cot \theta'_p \quad .$$

If $\theta_p > \frac{\pi}{2}$, then $\cos \theta_p < 0$, and

$$\frac{\beta_c}{\beta'_p} \csc \theta'_p + \cot \theta'_p < 0 .$$

This implies that

$$\left| \cos \theta'_p \right| > \frac{\beta_c}{\beta'_p}$$

and

$$\beta'_p > \beta_c \quad .$$

We conclude that, if the angle of the track is to exceed 90^o, its velocity in the center of mass system must exceed the laboratory velocity of the center of mass.

It can be shown(10) that the maximum center of mass velocity a particle can achieve in an interaction occurs when all other particles involved in the interaction move off as one body in the opposite direction. Thus, the maximum velocity of the proton in the center of mass system occurs when all the pions in the interaction move off together. Therefore, we have

$$P \longleftarrow \!\!\bullet\!\! \longrightarrow n\pi$$

c.m.

where $n\pi$ refers to n pions moving together as one body. Now

$$P'^2_p = P'^2_{n\pi} \quad .$$

Then

$$E'^2_p - m_p^2 = E'^2_{n\pi} - n^2 m_\pi^2 \tag{6}$$

If one squares the equation

$$E'_T = E'_p + E'_{n\pi}$$

where E'_T is the total energy in the center of mass system, one obtains

$$E'^2_{n\pi} = E'^2_T - 2E'_T E'_p + E'^2_p \quad . \tag{7}$$

Substituting Eq. (6) into Eq. (7) one gets

$$E'_p = \frac{E'^2_T + m_p^2 - n^2 m_\pi^2}{2E'_T} \quad .$$

This implies that E'_p, hence β'_p , is maximum when n is one.

For the case of one pion β'_p is constant and is given by

$$E'_p = \gamma'_p m_p = \gamma_c (E_p - \beta_c P_p) = \gamma_c m_p \quad .$$

Therefore, the maximum velocity of the proton in the center of mass system is given by

$$\gamma'_p = \gamma_c \quad .$$

Thus, the velocity of the proton in the center of mass system cannot exceed the velocity of the center of mass, and the angle of the proton track in the laboratory system cannot exceed 90^{o}.

It is believed that these selection rules have enabled us to obtain a group of events the majority of which are either interactions of pions with free protons or with protons which are loosely bound in nuclei.

Measurements

The desired information to be obtained from examination of the events consisted of momentum and space angle for each particle track. Space angle refers to the laboratory angle between a particle track and the direction of the incident pion. It is determined from the relation

$$\cos \theta = \cos \phi \cos \psi$$

where θ = space angle, ϕ = projected angle and ψ = dip angle. Projected angle refers to the projection of the space angle onto a plane perpendicular to the line of sight, while dip angle refers to the projection of the space angle of the track onto a plane through the track and perpendicular to the plane of the emulsion.

Angle measurements were accomplished in two steps. The projected angle was measured using an eyepiece goniometer constructed in the physics department shop. Accuracy of this goniometer is estimated to be $\pm$ 1 minute of arc. Dip angle was determined by measuring its tangent, which is the ratio of the true change in depth of a track segment to the length of the segment projected on the plane of the emulsion

plate. The measured change in depth was accomplished using a micrometer attached to the focusing mechanism of the microscope. The accuracy of this micrometer is $\pm$ 0.2 microns. Micrometer readings were noted while focusing first on one end and then on the other end of the chosen length of track. The difference between the two readings gave the measured change in depth of the track. This measured change in depth must be corrected for the shrinkage of the emulsion which occurred in the developing process. The average shrinkage factor was determined at Berkeley to be 2.372. The emulsions were kept at carefully controlled temperature and relative humidity to ensure a constant shrinkage factor. The true change in depth is then 2.372 times the measured change in depth.

Range measurements gave the momenta of stopping tracks while momenta for minimum ionizing tracks were determined by measuring the deflection of the tracks due to multiple Coulomb scattering. It was not possible to accurately measure the momentum of a minimum ionizing track whose length in the emulsion plate containing the interaction was less than 2.0 mm. However, scattering measurements were made on all tracks whose lengths were at least 2.0 mm in the plates containing their interactions. Noise elimination in the momentum calculations was accomplished by using a modified form of Barkas' method[11]. The modifications[12] concerned dropping Barkas' assumption that $\langle \delta_K \rangle = 0$ and realizing that difference products $\langle D^r_K D^r_{K+\ell} \rangle$ of all orders are not independent of one another. Using mean square averages of the independent second, third, and fourth differences, one obtains

$$\Delta_t^2 = \frac{2}{3}\left[\langle D^2_K\rangle + 2(\langle D_K D_{K+1}\rangle + \langle D_K D_{K+2}\rangle)\right]$$

as the mean square noise-corrected second difference. The equation used for mean absolute second difference assumes that second differences have a Gaussian distribution. Thus,

$$D = \left[\frac{2}{\pi}\Delta^2{}_t\right]^{\frac{1}{2}} \quad .$$

The standard cut-off at four times the average absolute second difference was used along with the corresponding scattering factor. $P\beta$ was then determined from

$$P\beta = \frac{K_c t^{3/2}}{573D}$$

where K_c is the dimensionless scattering factor, t is the cell length in microns, and 573 is a factor which gives units of MeV for $P\beta$ when D is measured in microns.

Scattering measurements were made using a base cell length of 200 microns. The number of second difference measurements varied for each track, usually being more than 10 and less than 100. Calculations for momenta were made at the base cell length and at every multiple thereof with the limitation that ten be the minimum number of measurements used for a calculation. The method of overlapping cells was used, giving one value of $P\beta$ at the measured cell length, two values at twice the cell length, etc., up to m values at m times the measured cell length, where m is limited by the number of measurements initially recorded. A single value of $P\beta$ was determined for each cell

length by averaging the different answers. The values for the different cell lengths were then compared. It was noted that these values varied from one cell length to another, usually becoming more consistent with increasing cell length. The value of $P\beta$ used in subsequent calculations was chosen from the values at the higher cell lengths which were in close agreement with one another. This selection was accomplished by comparing the standard deviations in the measured values at different cell lengths. The value of $P\beta$ with the least standard deviation was chosen as the best value.

In order to determine momenta from values of $P\beta$, one must assume masses for the minimum ionizing particles. In all subsequent calculations it was assumed that all the minimum ionizing particles were pions. Therefore, the mass of the pion was used for determining momentum from $P\beta$.

Errors

The error in each calculation of $P\beta$ was determined by considering that two-thirds of the scattering measurements used for calculation were independent[11] and then using the statistical estimation,

$$\Delta P\beta = \frac{P\beta}{\sqrt{\left(\frac{2}{3}\right)n}}$$

where n is the number of the measurements used for the calculation.

The error for the average $P\beta$ was calculated from

$$\overline{(P\beta)} = \frac{\left[\sum_{i=1}^{m} (\Delta P\beta)_i^2\right]^{\frac{1}{2}}}{m}$$

where m is the number of values used to calculate the average.

The error in space angle, θ, involved errors in measurement of projected angle, ϕ, and dip angle, Ψ. The error in measurement of ϕ was determined by repeatedly measuring a representative group of angles and then calculating the probable error for each angle in the group. The probable errors for these angles were very nearly the same, so their average of 0.004 radians was used for the error in each projected angle. Error in the measurement of a dip angle was due to the error in measuring the change in depth of the track. The average error determined by repeatedly measuring a representative set of tracks was found to be 0.6 microns. Space angle error was then determined from

$$\Delta\theta = \left[\left(\frac{\partial\theta}{\partial\phi}\Delta\phi\right)^2 + \left(\frac{\partial\theta}{\partial z}\Delta z\right)^2 \right]^{\frac{1}{2}}$$

$$= \csc\theta \left[(\sin\phi\cos\Psi\Delta\theta)^2 + (\cos\phi\sin\Psi\cos^2\Psi\Delta z/X)^2 \right]^{\frac{1}{2}} .$$

Errors in target mass were calculated from

$$\Delta M_{TGT} = \left[\left(\frac{\partial M_{TGT}}{\partial P_o}\Delta P_o\right)^2 + \left(\frac{\partial M_{TGT}}{\partial\beta_c}\Delta\beta_c\right)^2 \right]^{\frac{1}{2}}$$

$$= \left\{ \left[\left(\frac{1}{\beta_c} - \beta_o\right)\Delta P_o \right]^2 + \left[\left(\frac{P_o}{\beta_c}\right)\Delta\beta_c \right]^2 \right\}^{\frac{1}{2}}$$

where ΔP_o is 0.64 BeV, and $\Delta\beta_c$ for the effective target mass is given by

$$\Delta\beta_c = \left[\sum\left(\frac{\partial\beta_c}{\partial P_i}\Delta P_i\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial\theta_i}\Delta\theta_i\right)^2 \right]^{\frac{1}{2}}$$

$$= \left\{ \sum\left[(\cos\theta_i - \beta_c\beta_i)\Delta P_i \right]^2 + \sum\left[P_i\sin\theta_i\Delta\theta_i \right]^2 \right\}^{\frac{1}{2}}$$

for Methods I and III, and by

$$\Delta\beta_c = \left[\sum\left(\frac{\partial\beta_i}{\partial\theta_i}\Delta\theta_i\right)^2 + \sum\left(\frac{\partial\beta_i}{\partial P_T}\Delta P_T\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{\left\{\sum\left[\csc^2\theta_i(\beta_c\beta_i\cos\theta_i - 1)\Delta\theta_i\right]^2 + \sum\left[m_\pi^2\beta_c\beta_i\sin\theta_i\,\Delta P_T/P_T^3\right]^2\right\}^{\frac{1}{2}}}{\sum\left(\csc^2\theta_i + \frac{m_\pi^2}{P_T^2}\right)^{\frac{1}{2}}}$$

for Method II.

Error in β_c for total target mass is given by

$$\Delta\beta_c = \left[\sum\left(\frac{\partial\beta_c}{\partial P_i}\Delta P_i\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial P_n}\Delta P_n\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial\theta_i}\Delta\theta_i\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial\theta_n}\Delta\theta_n\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{f\sum E_i + \sum E_n}\left\{\sum\left[f(\cos\theta_i - \beta_c\beta_i)\Delta P_i\right]^2 + \sum\left[(\cos\theta_n - \beta_c\beta_n)\Delta P_n\right]^2\right.$$

$$\left. + \sum\left[f\,P_i\sin\theta_i\Delta\theta_i\right]^2 + \sum\left[P_n\sin\theta_n\Delta\theta_n\right]^2\right\}^{\frac{1}{2}}$$

for Methods I and III, and by

$$\Delta\beta_c = \left[\sum\left(\frac{\partial\beta_c}{\partial P_T}\Delta P_T\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial\theta_i}\Delta\theta_i\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial P_n}\Delta P_n\right)^2 + \sum\left(\frac{\partial\beta_c}{\partial\theta_n}\Delta\theta_n\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{f\,P_T\sum\left(\csc^2\theta_i + \frac{m_\pi^2}{P_T^2}\right)^{\frac{1}{2}} + \sum E_n}\left\{\sum\left[f(\cot\theta_i - \beta_c\beta_i\csc\theta_i)\Delta P_T\right]^2\right.$$

$$+ \sum\left[f\,P_T\csc^2\theta_i(\beta_c\beta_i\cos\theta_i - 1)\Delta\theta_i\right]^2 + \sum\left[(\cos\theta_n - \beta_c\beta_n)\Delta P_n\right]^2$$

$$\left. + \sum\left[P_n\sin\theta_n\Delta\theta_n\right]^2\right\}^{\frac{1}{2}}$$

for Method II.

Error in P_T was determined by the statistical estimation,

$$\Delta P_T = \frac{\overline{P}_T}{\sqrt{n}}$$

where n is the number of measurements used to calculate $\overline{P}_T$.

In Method III, ΔP_i for any particle with unmeasured momentum was determined from

$$\Delta P_i = \left[\left(\frac{\partial P_i}{\partial P_T} \Delta P_T\right)^2 + \left(\frac{\partial P_i}{\partial \theta_i} \Delta \theta_i\right)^2 \right]^{\frac{1}{2}}$$

$$= \left[(\csc \theta_i \Delta P_T)^2 + (P_T \csc \theta_i \cot \theta_i \Delta \theta_i)^2 \right]^{\frac{1}{2}} .$$

CHAPTER III

Results

Data

In scanning 514 meters of track, 60 events satisfying the criteria for pion-proton interactions were found. A total of 342 meters of track was scanned at Berkeley where 37 of the events were recorded and measured. The remaining events were recorded and measured here. There were 262 tracks associated with these events, of which 33 were heavily ionizing and 229 were minimum ionizing. The heavily ionizing tracks were assumed to be due to protons while the minimum ionizing tracks were assumed to be due to pions. The average number of minimum ionizing tracks per event was 4.37 ± 0.56. Figure 1 shows the distribution of the number of pion tracks per event. Data for all tracks is listed in TABLE 1.

Determination of Effective Target Mass

The methods developed in Chapter I for determining the effective target mass involve assumptions which can be compared with the experimental data. In Method I it was assumed that a representative set of pion tracks would be used in the determination of β_c. Since the equation for β_c involves the use of only those particles with directly

TABLE 1 DATA

EVENT	TRACK	$P\beta$ (MeV/c)	$\Delta P\beta$ (MeV/c)	$\theta(^\circ)$	$\Delta\theta(^\circ)$	$\phi(^\circ)$	$\psi(^\circ)$
15-38	1	946	259	9.6	0.2	7.4	6.1
	A			68.3	0.1	-50.0	-54.9
15-58	1			46.6	0.2	33.5	34.4
	2	4737	897	6.8	0.2	6.5	-1.9
	3	5987	1503	8.7	0.2	-8.0	3.4
	4			163.1	0.3	165.9	9.4
15-61	1			16.6	0.3	8.9	14.0
	2	479	147	9.6	0.3	6.6	7.0
	3	3959	996	3.2	0.3	-1.7	2.7
	4	3470	777	3.7	0.3	-2.6	2.6
	5			14.2	0.3	-9.3	10.8
	6			61.5	0.1	-22.8	-58.8
15-101	1	11696	2272	2.0	0.3	1.3	1.5
	A			77.0	0.2	-75.8	-24.1
15-118	1			22.6	0.2	13.8	18.0
	2	1415	388	12.1	0.2	10.8	5.4
	3	1425	280	2.1	0.2	1.6	-1.3
	4			10.1	0.3	-0.8	-10.1
	5			16.0	0.3	-1.4	-15.9
	6			20.6	0.3	-2.5	-20.5
15-127	1			19.4	0.3	8.4	17.5
	2	1530	382	5.1	0.2	3.8	3.5
	3	3944	1217	5.6	0.3	2.8	4.8
	4			5.0	0.3	-1.4	-4.8
	5	2350	798	10.0	0.2	-7.6	6.5
	6			31.4	0.2	-7.5	-30.5
	7			30.6	0.2	-23.4	-20.3
	8			150.8	0.2	115.1	-15.9
15-139	1			38.2	0.2	34.0	18.7
	2			12.5	0.3	2.7	-12.2
	3	7407	1608	3.3	0.3	1.6	2.9
	4			15.3	0.3	-9.4	12.2
	5	1055	145	11.8	0.2	-11.8	0.0
	6	233	76	33.2	0.2	-32.5	7.2
15-175	1	10623	1457	0.9	0.2	0.8	0.0
	A	71	7	81.6	0.2	-81.6	0.0
30-252	1	1875	439	19.3	0.2	19.1	2.8
	2			11.3	0.3	0.2	-11.3
	3	12016	1652	0.7	0.2	-0.7	0.0
	A	8	1	55.9	0.3	3.4	-55.9

EVENT	TRACK	$P\beta$ (MeV/c)	$\Delta P\beta$ (MeV/c)	$\theta(^\circ)$	$\Delta\theta(^\circ)$	$\phi(^\circ)$	$\psi(^\circ)$
30-291	1	13181	1703	0.6	0.2	-0.6	0.0
	A	10	1	57.6	0.2	56.8	-11.3
31-365	1			37.1	0.2	19.7	-32.0
	2	1009	292	8.3	0.2	-7.9	2.7
31-370	1	2146	560	7.0	0.2	6.2	3.4
	A	8	1	29.8	0.3	8.0	-28.8
31-380	1	4998	773	2.7	0.2	2.3	-1.5
	2	1413	480	5.0	0.3	1.2	4.8
	3			13.6	0.3	-2.9	13.2
	4			54.1	0.2	-45.8	-32.8
31-392	1	8561	1586	5.3	0.2	5.0	-1.8
	2	3129	495	8.9	0.2	-8.8	1.3
31-394	1	6727	2484	4.4	0.3	2.8	3.4
	2	1628	499	6.5	0.3	3.0	-5.8
	3	648	239	9.3	0.3	-1.0	-9.3
	4	745	323	5.0	0.3	-1.8	4.7
	5			20.4	0.3	-3.7	-20.0
	6			20.2	0.2	-17.0	11.0
24-451	1	213	87	12.2	0.3	7.6	-9.5
	2	3404	696	4.4	0.2	4.1	-1.6
	3			22.7	0.3	0.9	22.7
	4	4885	1545	6.2	0.2	-5.9	1.8
	5			86.0	0.2	-85.9	-11.5
	A	129	13	47.8	0.4	28.5	40.1
24-453	1	554	175	8.8	0.2	8.1	-3.3
	2	1237	325	6.0	0.2	5.6	-2.0
	3	619	253	10.4	0.3	5.0	9.1
	4	1683	486	0.5	0.2	0.5	0.0
	5	971	359	7.9	0.3	-1.1	-7.8
	6	2414	441	2.6	0.2	-2.6	0.0
	7			11.7	0.3	-5.[illegible]	10.1
	8	1429	450	25.7	0.2	-25.5	-3.3
	9			86.3	0.2	-85.9	25.5
	A	41	4	68.0	0.2	59.8	41.8
24-456	1	10772	1482	1.7	0.2	1.7	0.0
	A			76.6	0.2	-76.6	0.0
24-466	1	4136	566	2.3	0.2	1.8	1.4
	2	908	352	5.9	0.3	2.1	-5.5
	3			46.3	0.2	-45.6	-8.9
	A	176	18	8.0	0.2	8.0	0.0

EVENT	TRACK	P/β (MeV/c)	ΔP/β (MeV/c)	Θ(°)	ΔΘ(°)	φ(°)	ψ(°)
24-510	1	2372	484	2.5	0.3	0.9	-2.3
	2	3674	488	0.6	0.2	-0.6	0.0
	3	6232	1030	0.2	0.2	-0.2	0.0
	A			81.8	0.0	6.7	81.7
24-515	1	2193	549	4.0	0.3	0.2	4.0
	2			66.6	0.1	-55.7	45.2
24-546	1			25.6	0.2	18.9	17.6
	2			13.1	0.2	11.4	6.6
	3	280	95	9.0	0.3	5.2	-7.4
	4			7.8	0.3	3.4	7.0
	5	361	140	9.7	0.3	1.1	-9.7
	6	3062	395	4.3	0.2	-4.3	0.0
	7			17.5	0.2	-15.1	8.8
	8			35.2	0.2	-31.7	16.2
35-606	1	1061	161	24.0	0.2	24.0	0.0
	2			21.0	0.2	19.5	8.0
	3	3514	635	5.0	0.2	4.4	-2.3
	4	776	287	9.6	0.3	-2.5	-9.3
41-15	1	5000	580	1.4	0.2	0.8	1.1
	A	110	11	74.4	0.2	-69.7	-39.3
41-18	1	6000	695	0.0	0.2	0.0	0.0
	2	504	150	2.6	0.2	-2.3	1.1
41-35	A	153	15	56.1	0.2	49.4	31.1
	2	3380	470	2.0	0.2	-0.9	-1.7
	3			6.6	0.2	-2.1	-6.3
	4			69.0	0.2	-64.0	35.3
41-36	1	2110	361	3.6	0.2	3.5	0.6
	2	6160	695	1.4	0.2	1.0	-0.9
	3	5210	755	5.4	0.2	-5.1	-1.8
	A	31	3	73.3	0.2	73.0	-10.5
41-38	1			29.9	0.2	28.7	-8.9
	2	2810	406	2.8	0.2	-2.4	-1.4
	3			12.4	0.2	-3.4	11.9
	4			16.8	0.2	-12.7	-11.0
42- 3	1			16.0	0.2	7.9	-14.0
	A	300	90	3.4	0.2	2.6	-2.2
	3	7010	2600	0.1	0.2	-0.1	0.0
	4	1320	163	7.4	0.2	-7.2	-1.5

EVENT	TRACK	Pβ (MeV/c)	ΔP/β (MeV/c)	Θ(°)	ΔΘ(°)	φ(°)	ψ(°)
56-19	1	323	140	13.8	0.2	13.6	-2.4
	2			25.8	0.2	9.8	24.0
	3			3.5	0.2	2.9	-1.9
	4	172	67	9.3	0.2	-7.4	-5.7
	5	482	139	23.2	0.2	-22.0	-7.7
	6			42.6	0.2	23.4	-36.7
56-32	1	5140	611	1.9	0.2	-1.4	1.2
	A	196	20	76.6	0.2	76.3	11.1
56-53	1			20.0	0.2	8.5	18.2
	2	1830	648	4.9	0.2	3.4	3.6
	3	1280	327	2.4	0.2	1.8	1.6
	4			8.2	0.2	-3.1	7.6
	5	2600	712	5.3	0.2	-5.0	-1.8
	A	193	19	47.4	0.2	-18.3	-44.5
56-57	1			14.9	0.2	12.8	-7.7
	2	1810	435	2.3	0.2	2.2	-0.6
	3			20.6	0.2	-4.0	20.3
	4			27.4	0.2	-27.1	-4.4
65-15	1			12.5	0.3	9.8	-7.8
	2			6.5	0.4	-2.1	-6.1
	3	4460	1092	5.8	0.2	-5.6	1.4
	4	204	53	5.8	0.2	-5.6	1.4
	5	1060	324	10.7	0.2	-7.9	7.2
	A	69	7	59.0	0.2	46.2	41.9
69- 2	1			9.5	0.3	-8.2	-4.8
	2			10.9	0.4	-7.0	-8.4
	3			9.9	0.4	-7.0	-7.0
	4			39.7	0.3	35.9	-18.3
69-14	A	311	31	29.7	0.3	-21.4	21.1
	2	7500	1965	2.7	0.5	-0.6	-2.6
	3	1500	278	7.8	0.2	7.5	2.0
	4	1790	694	9.8	0.3	7.8	6.0
69-16	1	1910	585	15.8	0.4	-7.4	14.0
	2	1300	206	5.3	0.3	-4.9	-2.0
	3			11.4	0.4	-2.0	11.2
	4			12.6	0.5	0.3	-12.6
	5			8.3	0.5	0.4	-8.3
	6			11.3	0.5	0.6	11.2
	7			12.5	0.4	5.9	-11.1
	A	7	1	16.7	1.8	-10.9	12.8

EVENT	TRACK	$P\beta$ (MeV/c)	$\Delta P\beta$ (MeV/c)	Θ(°)	$\Delta\Theta$(°)	ϕ (°)	ψ (°)
69-58	1	2790	441	5.9	0.2	-5.5	2.0
	2	2940	670	4.1	0.4	2.0	3.6
69-64	1			5.0	0.4	-2.7	4.2
	2			28.0	0.3	18.8	-21.1
	3			155.9	0.4	175.9	-23.7
	4			46.6	0.2	-16.2	-44.4
76-35	1	3310	907	1.4	0.2	-1.4	0.0
	A	55	6	70.9	0.3	-6.5	70.8
76-42	1	277	128	12.4	0.2	8.0	9.6
	2			110.1	0.3	-117.6	-42.1
76-54	1			12.7	0.5	-3.2	-12.3
	2			9.9	0.5	0.5	9.9
	3			7.2	0.5	0.8	-7.2
	4	2430	413	11.4	0.2	11.1	2.8
	5			16.7	0.3	11.9	11.8
	6			22.9	0.4	5.3	-22.3
76-91	1			29.2	0.2	-27.7	-9.5
	2	3730	895	2.3	0.4	1.2	2.0
	3	363	158	41.6	0.2	41.4	-4.2
	4			32.6	0.3	-20.1	-26.2
76-109	1			39.0	0.3	-19.4	34.5
	2	1010	149	5.1	0.2	5.1	0.0
	3	564	151	19.4	0.2	19.3	-1.5
	4			116.2	0.2	116.5	-8.7
76-125	1			21.0	0.4	-12.4	17.1
	A	128	13	45.3	0.2	8.7	-44.6
76-143	1	3020	516	1.5	0.2	1.5	0.0
	2	5310	610	3.8	0.3	3.4	1.6
	3			38.5	0.2	36.7	-12.4
	A	8	1	62.1	0.3	61.5	-10.9
76-154	1			45.1	0.3	-29.1	36.1
	2	2310	423	4.4	0.3	-3.8	2.2
	3			6.1	0.5	-1.0	-6.0
	4	2380	278	1.2	0.2	1.2	0.0
	5	6370	1270	7.0	0.2	7.0	0.0
	6			61.1	0.2	-59.4	18.3

EVENT	TRACK	$P\beta$ (MeV/c)	$\Delta P\beta$ (MeV/c)	$\Theta(^{o})$	$\Delta\Theta(^{o})$	$\phi(^{o})$	$\psi(^{o})$
78-10	1	1090	403	19.4	0.2	18.2	-6.8
	2			32.5	0.2	12.2	-30.4
	3	659	190	8.8	0.2	8.7	1.6
	4	3980	1175	4.3	0.2	3.1	-3.0
	5			40.7	0.2	-38.0	-15.9
	A	0.6	0.1	40.0	0.2	-40.0	-1.4
78-71	1			41.1	0.2	40.2	9.5
	2	635	178	31.4	0.2	31.4	1.8
	3	5570	1202	7.1	0.2	6.6	-2.5
	4	4250	1340	7.1	0.2	3.7	-6.1
	5			19.0	0.2	-15.8	10.8
	6	1040	368	32.1	0.2	-31.6	-5.9
	7	741	111	35.8	0.2	-35.8	-1.4
	A	186	19	67.4	0.2	-63.4	-30.8
78-79	1			73.3	0.2	69.0	36.6
	2			31.0	0.2	30.5	6.0
	3	1780	254	8.6	0,2	8.4	-2.0
	4	5460	2360	8.9	0.2	7.5	-4.8
	5	2110	346	2.4	0.2	-2.2	-1.0
	6	828	207	10.6	0.2	-9.7	-4.2
	7			19.1	0.2	-11.4	-15.4
	8			45.4	0.2	-44.1	12.1
78-107	1			11.2	0.2	-0.2	-11.2
	2			15.9	0.2	-6.3	-14.7
	3	5570	1970	10.1	0.2	-9.9	-2.1
	A	13	1	57.8	0.4	53.9	25.2
78-112	1	6000	1134	2.6	0.2	1.9	1.7
	2	3320	650	1.7	0.2	-0.8	1.5
	A	0.8	0.1	36.6	0.3	-5.9	-36.2
	4			25.2	0.2	-12.8	-22.0
78-136	1	3050	394	6.7	0.2	6.6	1.0
	A	43	4	24.9	0.9	-15.8	-19.6
78-138	1	3460	560	4.0	0.2	3.6	-1.6
	2	2300	851	4.7	0.2	3.0	3.6
	3	7600	2690	4.7	0.2	2.3	-4.1
	4			54.5	0.2	-45.2	34.6
78-140	1	4480	2080	13.5	0.2	11.3	7.4
	2			26.2	0.2	-3.5	25.9
	3			78.9	0.1	30.2	77.1
	A	0.7	0.1	53.6	0.2	53.6	0.0

EVENT	TRACK	P/β (MeV/c)	$\Delta P/\beta$ (MeV/c)	$\Theta(^\circ)$	$\Delta\Theta(^\circ)$	$\phi(^\circ)$	$\psi(^\circ)$
85-6	1	5450	2020	5.3	0.2	-4.9	-2.0
	2	2460	1230	2.6	0.2	-1.8	1.8
	3	1330	615	7.4	0.2	-5.1	-5.4
	A	140	14	13.2	0.2	11.1	-7.3
85-41	1			27.0	0.2	20.2	18.3
	2			24.3	0.2	15.9	-18.7
	3			14.8	0.2	10.3	-10.6
	4	666	308	9.1	0.2	8.9	1.7
	5	2670	697	2.8	0.2	2.3	-1.5
	6	859	351	16.4	0.2	-14.6	7.5
	7			25.7	0.2	-22.7	12.5
	8			51.6	0.2	-47.1	-24.0
	9			51.1	0.2	-50.8	-6.8
	A			72.0	0.1	-10.3	71.7
85-52	1			26.2	0.2	24.9	-8.4
	2	1220	610	10.8	0.2	9.1	5.8
	3	2190	650	9.3	0.2	-9.1	2.1
	4			49.8	0.2	-47.8	-16.3
85-55	1			149.0	0.2	176.0	-30.7
	2			17.8	0.2	10.1	-14.7
	3			9.2	0.2	4.2	-8.2
	4	627	290	10.2	0.2	-3.6	9.5
	5	4260	1970	10.6	0.2	-10.1	3.2
	A	0.9	0.1	56.2	2.6	-48.4	33.0
100-45	1			15.2	0.2	4.0	14.6
	2	5390	1045	5.0	0.2	3.0	-4.1
	3			41.8	0.3	-27.1	33.2
	A	56	6	63.5	0.3	7.1	-63.3

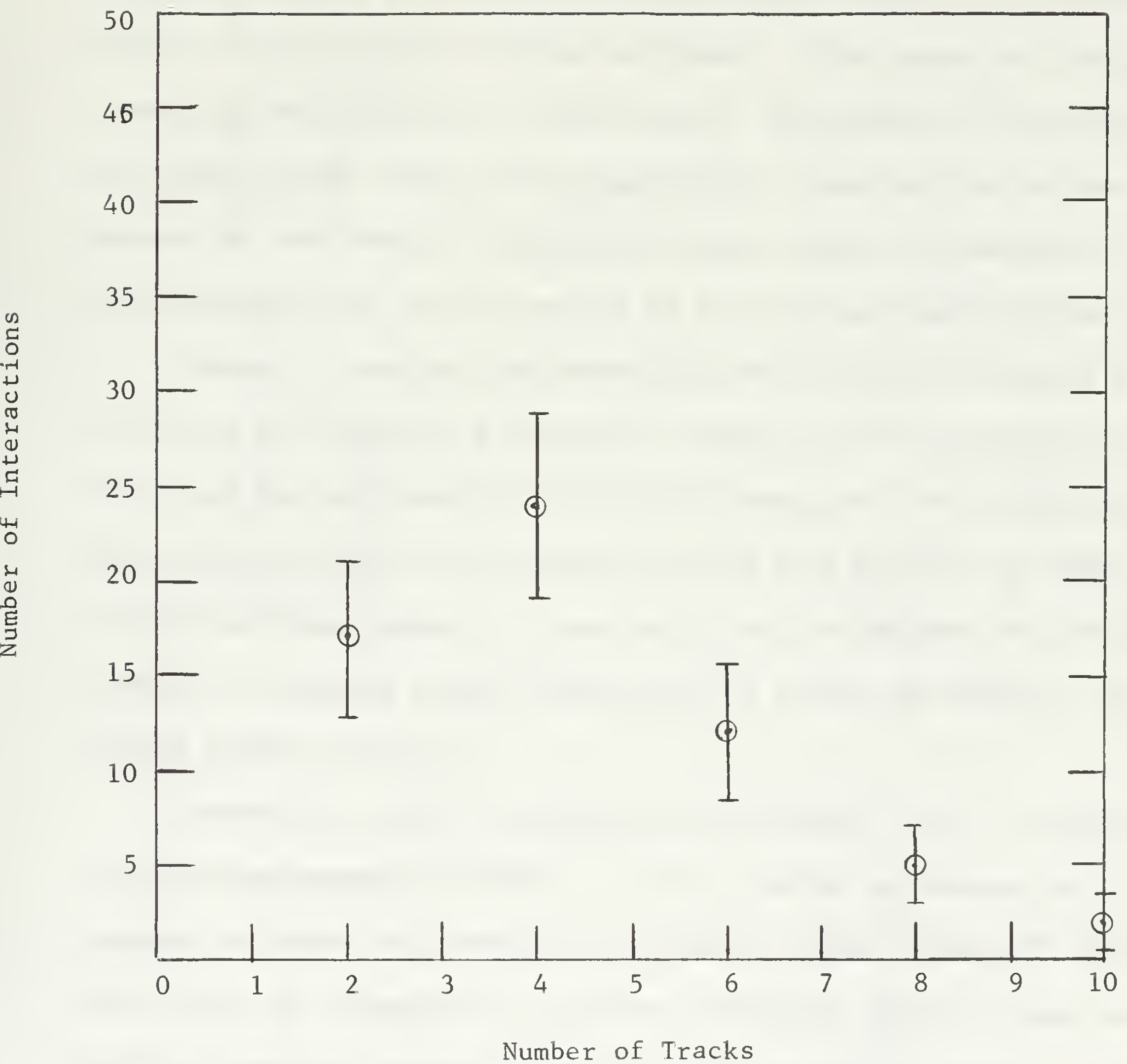

Fig. 1. Distribution of Tracks for Pion-Proton Interactions

measured momenta, one must compare the angular distribution of the tracks with measured momenta with that of the entire set of tracks to determine whether or not the set with measured momenta is representative. This comparison is shown in Figure 2. One notes that the distributions are dissimilar in that most of the momenta of the tracks in the lower angular range were measured while relatively few of the momenta of the tracks in the higher angular range were measured. Thus, one concludes that the set may not be a truly representative set.

Method II involves the assumption that transverse momenta of the pions can be treated as a constant. Figure 3, which represents our data, and the references(1),(2),(3),(4) mentioned above indicate that $\overline{P}_T$, which was used as the constant, varies with angle up to about 10^o-15^o and then appears to level off. Thus, it appears that the assumption is invalid at small angles but is a fair approximation at angles greater than 10^o

Method III, which is combination of Methods I and II, would seem to be an improvement on either. It was possible to measure the momenta of almost all tracks in the angular region between 0^o and 10^o and, since the assumption of constant transverse momentum seems valid for the remaining angular region, it would appear that the possible discrepancies in Methods I and II have been eliminated.

Average transverse momenta for Method III were calculated from tracks with measured momenta to be 177 ± 25 MeV/c for the angular region between 0^o and 5^o, 297 ± 44 MeV/c for the angular region between 5^o and 10^o, 348 ± 87 MeV/c for the angular region between 10^o and 15^o, and

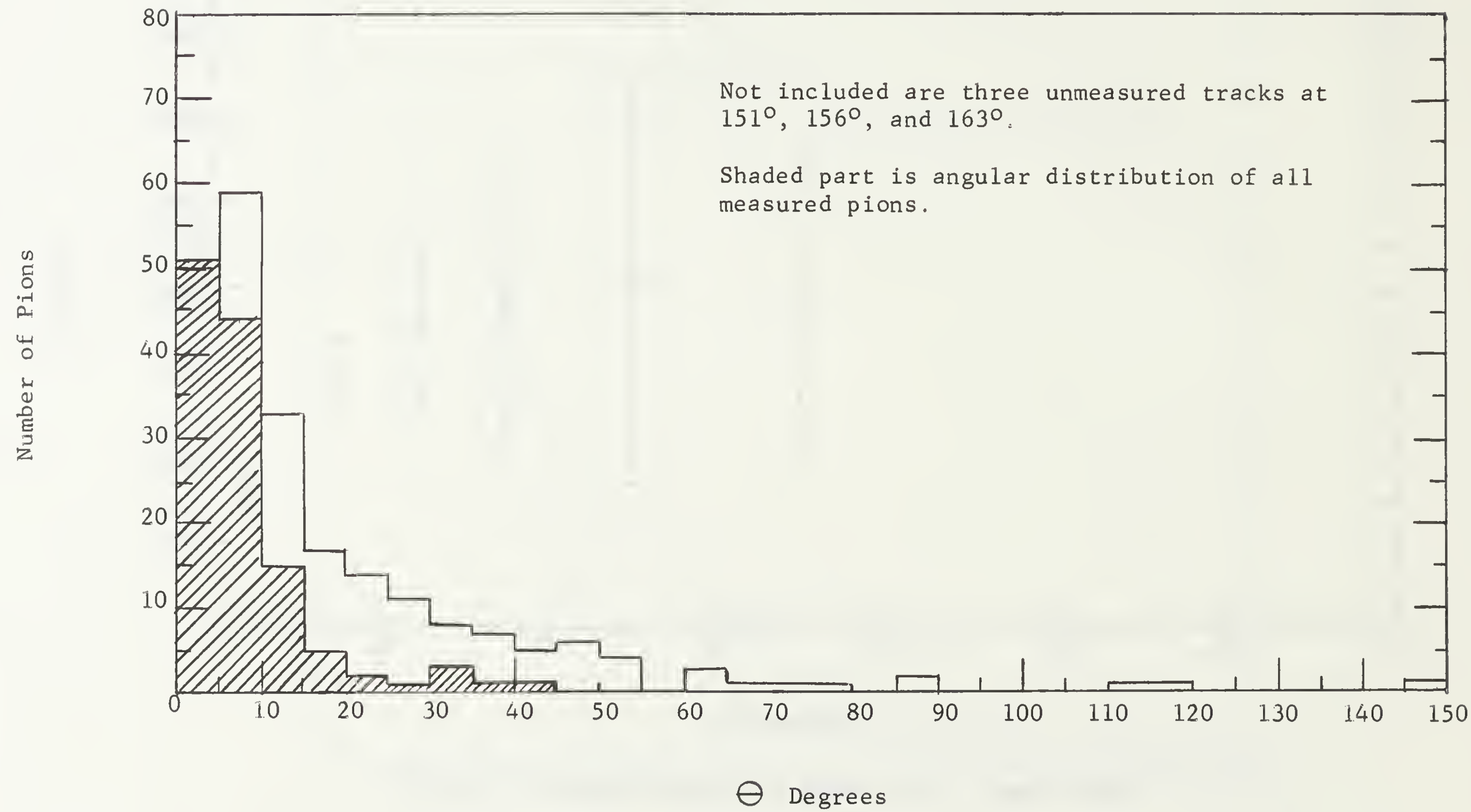

Fig. 2. Angular Distribution of All Pions

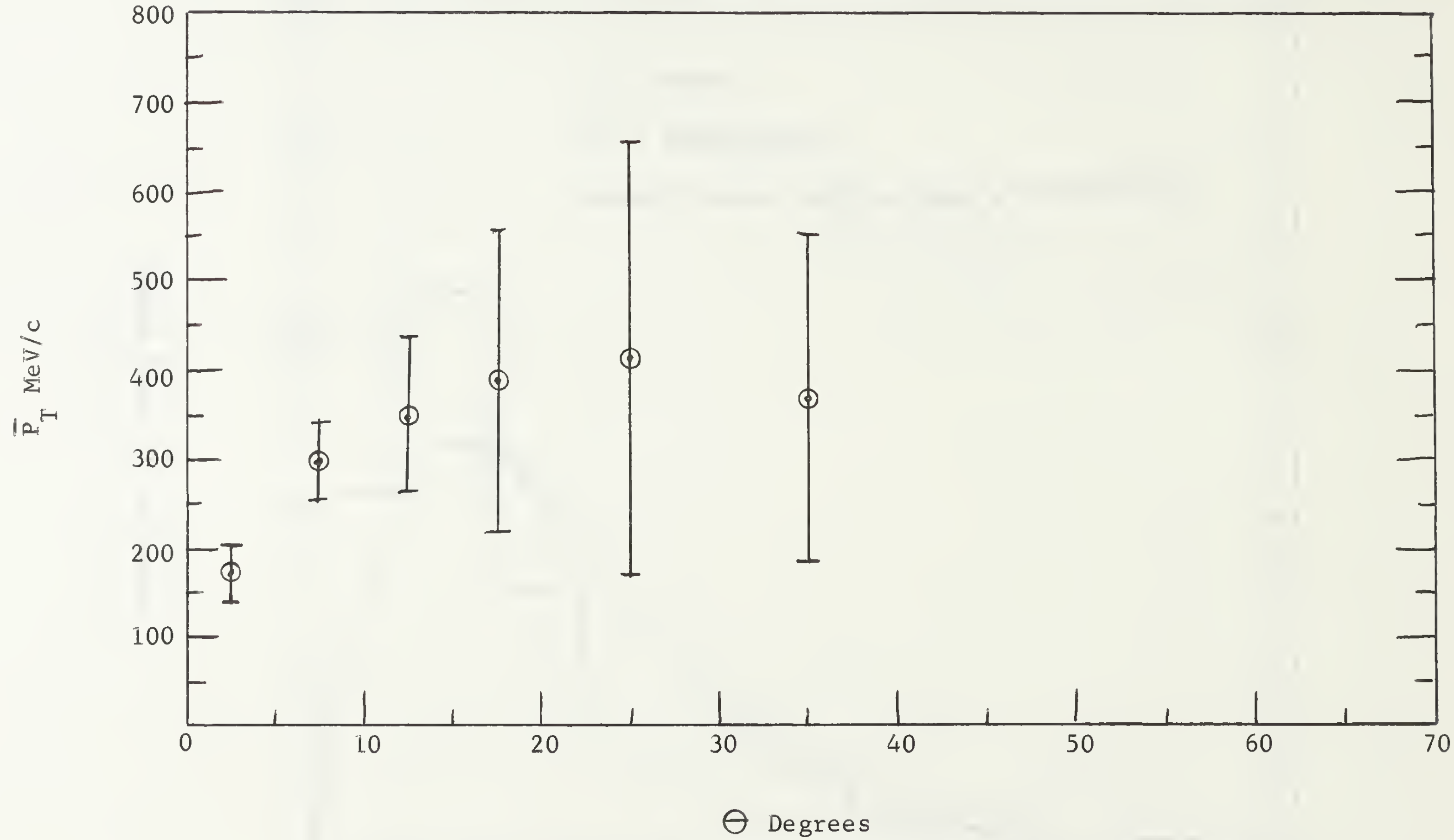

Fig. 3. Average Transverse Momentum vs. Space Angle

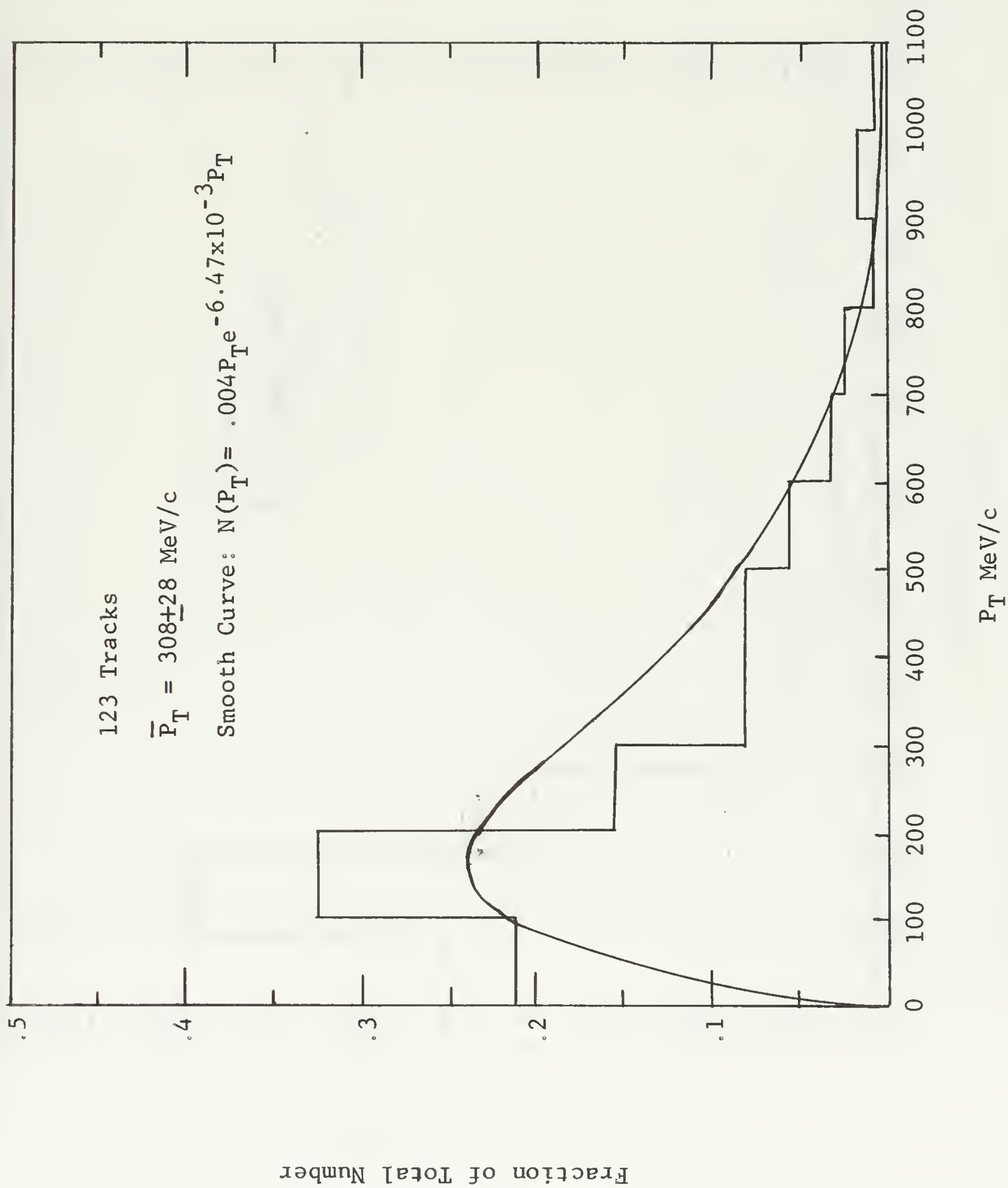

Fig. 4. Transverse Momentum Distribution for All Pions

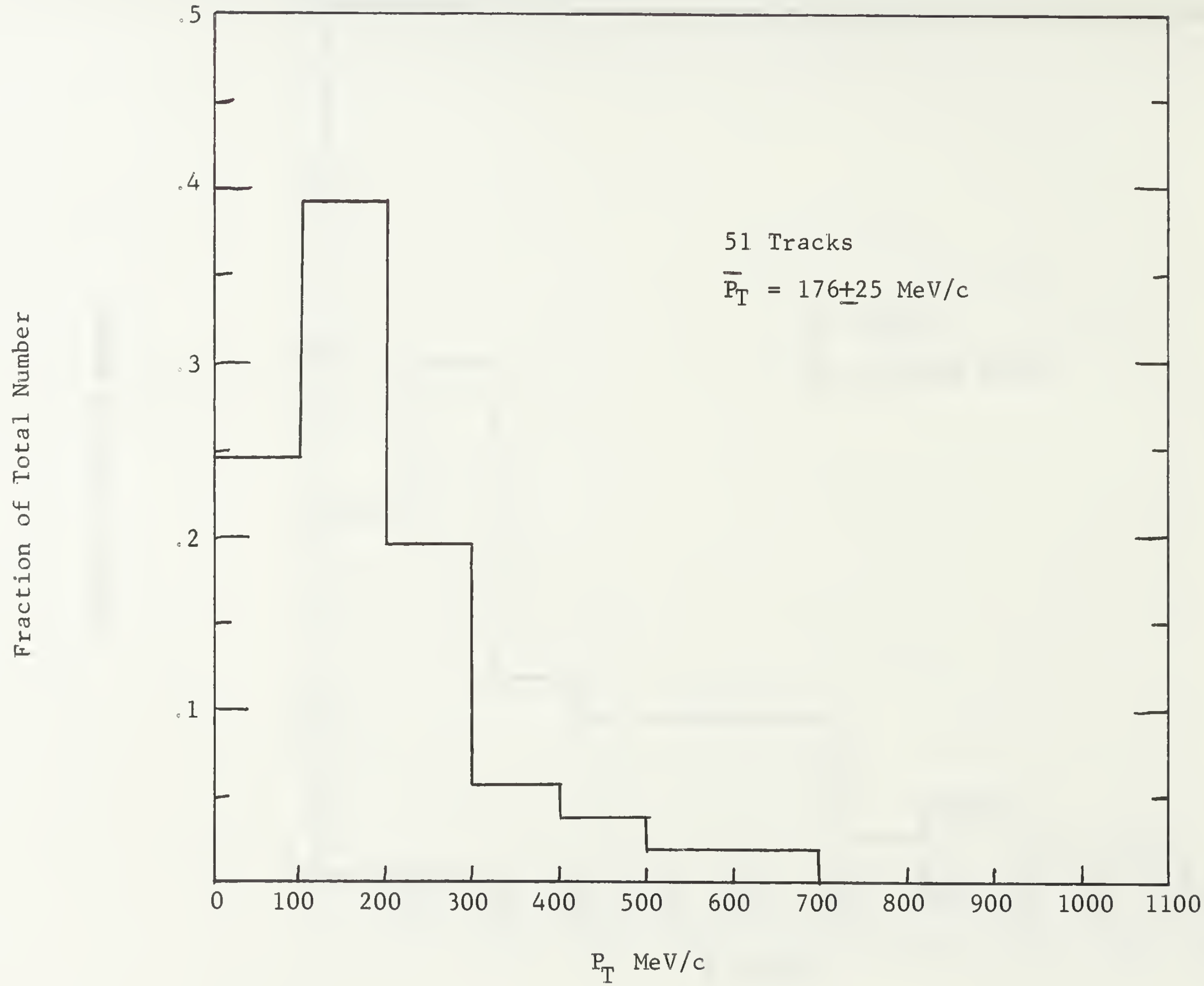

Fig. 5. Transverse Momentum Distribution for Pions with $0^\circ \leq \theta \leq 5^\circ$

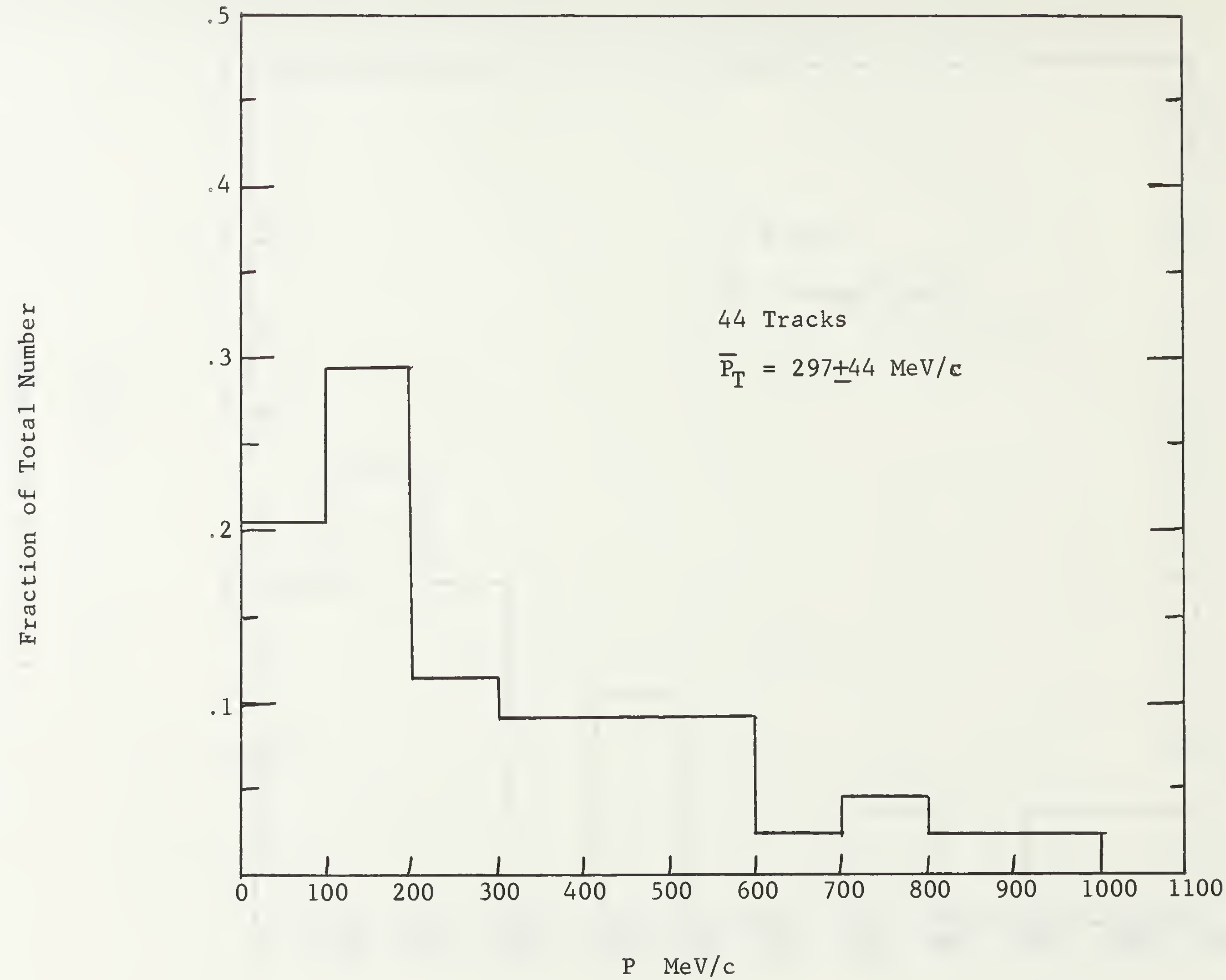

Fig. 6. Transverse Momentum Distribution for Pions with $5^\circ < \theta \leq 10^\circ$

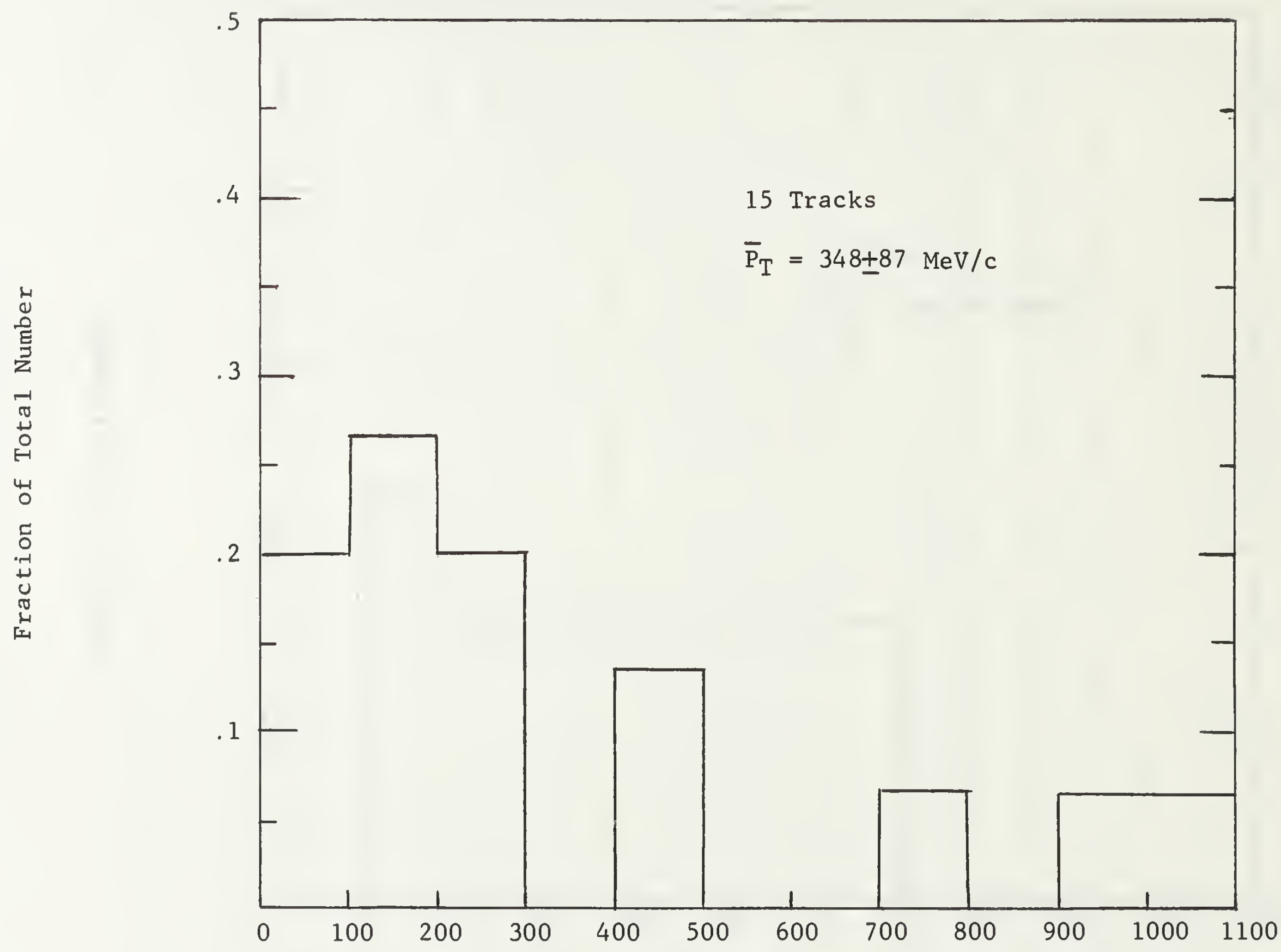

Fig. 7. Transverse Momentum Distribution for Pions with $10^{\circ} < \theta \leq 15^{\circ}$

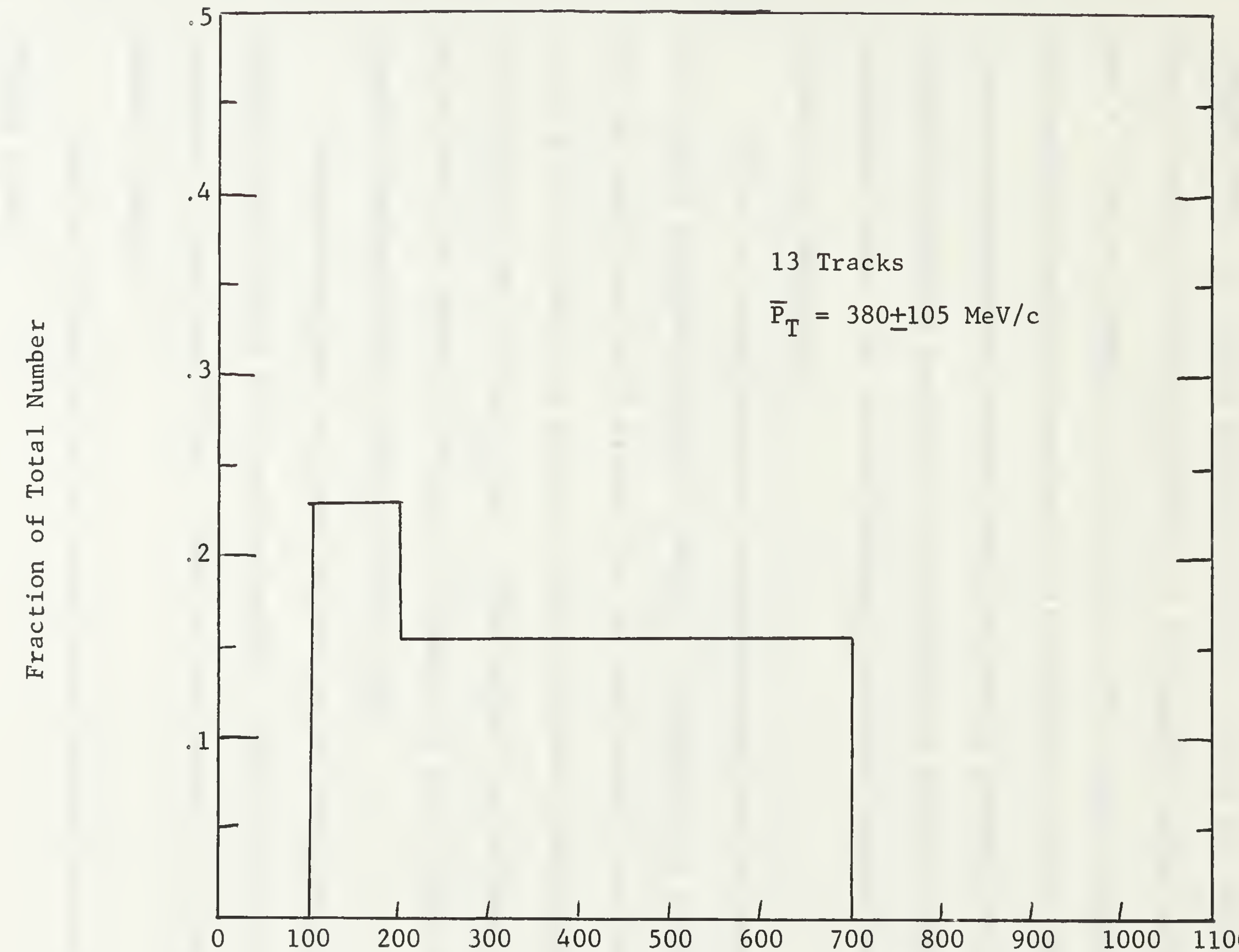

Fig. 8. Transverse Momentum Distribution for Pions with $15^{\circ} \angle \theta$

380 ± 105 MeV/c for angles greater than 15°. It was not necessary to use an average transverse momentum in the angular region between 0° and 5° since the momenta of all tracks in this region were measured. The average transverse momentum of all tracks was used for Method II. Tracks with unmeasured momenta were included by assuming the average measured transverse momentum in the appropriate angular range for each unmeasured track. The overall average transverse momentum was calculated to be 308 ± 28 MeV/c.

Effective target mass was calculated using each of these methods for several different groups of events. In addition to a calculation for the entire set of 60 events, the set was divided into groups according to the number of observed tracks per event, and calculations were made for the various groups. These groups are defined in TABLE 2, which shows the number of events, the number of pion tracks and the number of measured pion tracks for each group.

Calculations were also made for the set of events which had low-energy recoil protons with measured momenta. These protons had kinetic energies of less than 175 MeV. This set was divided into groups as shown in TABLE 3.

Results of these effective target mass calculations are found in TABLES 4 and 5.

Determination of Total Target Mass

As indicated in Chapter I weighting factors must be introduced into total target mass calculations to obtain a representative set of secondary tracks. The weighting factors for the three methods are

TABLE 2

Division of Groups of Events for the Entire Set

# Tracks/Event	2,4	6,8,10	2	4	6	8
# Events	41	19	17	24	12	5
# Pions	106	123	23	83	67	38
# Measured Pions	63	60	19	44	33	17

TABLE 3

Division of Groups of Events

for the Set With Low-Energy Recoil Protons

# Tracks/Event	All	2,4	6,8,10
# Events	28	20	8
# Pions	92	44	48
# Measured Pions	58	30	28

TABLE 4

Effective Target Mass in MeV/c^2 for the Entire Set of Interactions

# Tracks Per Event	METHOD I M_{TGT}	METHOD I ΔM_{TGT}	METHOD II M_{TGT}	METHOD II ΔM_{TGT}	METHOD III M_{TGT}	METHOD III ΔM_{TGT}
All	156	8	209	469	761	49
2,4	93	6	130	359	650	59
6,8,10	284	18	552	18	925	65
2	50	4	27	134	231	26
4	118	9	265	182	858	87
6	212	14	510	23	757	68
8	389	36	905	38	1140	132

TABLE 5

Effective Target Mass in MeV/c^2 for the Set of Interactions with Low-Energy Recoil Protons

# Tracks Per Event	METHOD I M_{TGT}	METHOD I ΔM_{TGT}	METHOD II M_{TGT}	METHOD II ΔM_{TGT}	METHOD III M_{TGT}	METHOD III ΔM_{TGT}
All	160	11	169	111	478	36
2,4	74	8	74	63	243	17
6,8,10	379	31	486	59	941	97

determined from various considerations. For each group one must normalize the number of pions used for calculations to the total number of pions emitted in the events included in the group, and one must perform a similar normalization for protons. Let f_1 be the factor which normalizes pion sums and f_2 be the factor which adjusts proton sums. Then the weighting factor used in Chapter I is given by

$$f = \frac{f_1}{f_2} \ .$$

One will, on the average, observe only two thirds of the emitted pions since charge independence of nuclear forces implies that about one third of the emitted pions should be neutral and will leave no tracks in the emulsion. Then f_1 will be 3/2 for all groups in Methods II and III in which all observed pions are used for calculations. Method I uses only pions with measured momenta and, therefore, another factor in addition to 3/2 must be introduced in determining f_1 for this method. f_1 becomes

$$f_1 = \left(\frac{3}{2}\right) \left(\frac{N_t}{N_m}\right)$$

where N_m is the number of tracks with measured momenta and N_t is the total number of observed tracks.

Conservation of baryons demands that (unless $p\,\bar{p}$ production occurs) one and only one nucleon be emitted in each interaction. Although proton tracks can be observed in the emulsion, momenta were not determined for all proton tracks. Events without an identifiable proton were assumed to include a neutron with an average energy and angular

distribution equal to those of the measured protons. This assumption was made because all identifiable protons had kinetic energies less than 175 MeV. Although protons with kinetic energies up to at least 500 MeV should be readily identifiable, none with energies between 175 MeV and 500 MeV were observed. Accounting for these considerations, f_2 becomes

$$f_2 = \frac{N_e}{N_p}$$

where N_p is the number of protons with measured momenta and N_e is the number of events considered in the calculations. The factor f becomes

$$f = \left(\frac{3}{2}\right) \left(\frac{N_t}{N_m}\right) \left(\frac{N_p}{N_e}\right)$$

for Method I and

$$f = \left(\frac{3}{2}\right) \left(\frac{N_p}{N_e}\right)$$

for Methods II and III.

These factors are listed in TABLES 6 and 7 for each group of events for which a calculation of M_{TGT} was made. Results of the calculations of total target mass are listed in TABLES 8 and 9.

TABLE 6

Weighting Factors for the Entire Set of Interactions

# Tracks/Event	All	2,4	6,8,10	2	4	6	8
Method I	1.30	1.23	1.29	0.85	1.41	1.27	1.34
Methods II & III	0.70	0.73	0.63	0.71	0.75	0.63	0.60

TABLE 7

Weighting Factors for the Set of Interactions

with Low-Energy Recoil Protons

# Tracks/ Event	All	2,4	6,8,10
Method I	2.38	2.20	2.57
Methods II & III	1.50	1.50	1.50

TABLE 8

Total Target Mass in MeV/c^2 for the Entire Set of Interactions

# Tracks Per Event	METHOD I M_{TGT}	METHOD I ΔM_{TGT}	METHOD II M_{TGT}	METHOD II ΔM_{TGT}	METHOD III M_{TGT}	METHOD III ΔM_{TGT}
All	920	43	571	*	1859	90
2,4	943	47	432		1891	105
6,8,10	969	57	1180		1819	103
2	1457	88	260		1883	115
4	797	45	655		1890	125
6	926	66	1228		1681	110
8	957	86	1655		1921	171

*Errors for Method II were, in general, much larger than the values for M_{TGT}. This is a result of the problem discussed in Chapter IV.

TABLE 9

Total Target Mass in MeV/c^2 for the Set of Interactions with Low-Energy Recoil Protons

# Tracks Per Event	METHOD I		METHOD II		METHOD III	
	M_{TGT}	ΔM_{TGT}	M_{TGT}	ΔM_{TGT}	M_{TGT}	ΔM_{TGT}
All	970	51	634	*	1543	82
2,4	943	54	502		1378	77
6,8,10	1162	87	1068		1869	141

*Errors for Method II were, in general, much larger than the values for M_{TGT}. This is a result of the problem discussed in Chapter IV.

CHAPTER IV

Conclusions

The question of which of the three methods of determining effective target mass is more correct is answered by a study of TABLES 8 and 9 and an examination of the effects of the assumptions made in Methods II and III. A correct method of determining target mass should yield values of total target mass consistent with the mass of the proton, 938 MeV/c^2, if selection of events was accomplished properly. Except for the group of events with two tracks per event, which yields a mass value of 1457$\pm$88 MeV/c^2, the values of total target mass determined by Method I are all very close to the mass of the proton. The disagreement in the two-track group can be explained by a closer examination of the events used for this calculation of target mass. One can see by examining the data in TABLE 1 that events 15-101, 15-175, 30-291, 24-456, and 41-15 have coplanar tracks, one of which is a pion track, the other a proton track. This implies that these events are probably elastic collisions. The momenta of the protons in three of the events were determined from angles and the assumption that the collisions are elastic. The momentum values obtained in this way agree, within error, with the values determined by range measurements. This gives further evidence that the events are elastic collisions.

Momenta of the protons in the remaining events were not measured by the range method and, therefore, a comparison is not possible.

If one assumes that these events are elastic collisions, the momenta of the secondary pions can be determined more accurately by measuring only the angle of the proton track. Calculations show that all pions in these events have momenta of 16.2 ± 0.8 BeV/c if the collisions are elastic. One notes that this value is significantly higher than the value calculated by scattering methods for each of the pions. It was noted in selecting $P\beta$ for these particles that the values determined by scattering at different cell lengths varied considerably. Thus, the choice of which $P\beta$ to use was not as precise as in most cases.

Events 31-392 and 41-18 also have coplanar tracks, but both tracks are minimum ionizing. If one assumes that these events are elastic collisions, then one of the minimum ionizing tracks in each event must be due to a proton.

The weighting factor, f, used in the determination of β_c for this group should be adjusted to account for these events. Calculation of total target mass for this group considering that these collisions were elastic yields a value of 903 ± 55 MeV/c. It appears then, that although discrepancies occur in the angular distribution, the set of tracks with measured momenta is representative of the entire set and that Method I is a reliable method of calculating effective target mass.

The values of total target mass determined by Method II are, with few exceptions, much less than the mass of the proton. Since the minimum acceptable total target mass is the proton mass, this indicates that Method II is unacceptable as a means of calculating target mass. The fault in the method almost certainly lies in the assumption of constant transverse momenta for the region of small angles. One can see from Figure 3 and the earlier references [(1),(2),(3),(4)] that average transverse momentum is definitely not constant in the angular region between 0^o and 10^o. If one considers the fact that the cotangent and the cosecant approach infinity as space angle approaches 0^o, then it is not surprising that total target mass is underestimated by Method II since, for this method,

$$\beta_c = \frac{f\ P_T \sum \cot\ \theta_i + \sum P_n \cos\ \theta_n}{f\ P_T \sum (\csc^2\ \theta_i + \frac{m}{P_T{}^2})^{\frac{1}{2}} + \sum E_n}$$

which approaches unity if any $\theta_i \longrightarrow 0^o$. Therefore, under the same circumstance,

$$M_{TGT} = \frac{P_o}{\beta_c} - E_o \longrightarrow P_o - E_o \simeq 0 \quad .$$

There are, in fact, several tracks with very small angles in the total group and it is due to these tracks that total target mass is underestimated by Method II. It should be noted that the tracks with small angles do not have as much effect on the calculations of effective target mass since in this case β_c is close unity even if the tracks with small angles are neglected.

Method III, which seems to be an improvement on Methods I and II, yields values of total target mass which are approximately twice the mass of the proton. Here again the fault may lie in the assumption of constant transverse momentum. The momenta of tracks with small angles were directly measured in Method III and, therefore, the problem discussed above for Method II is not a factor. However, closer examination of Figure 3 and the work of others[(1),(2),(3),(4)] reveals that small statistics are used in the determination of the average transverse momentum in the angular region between 15° and 180°. In this paper, the average transverse momentum for the 86 tracks with angles between 15° and 163° was determined using the measured transverse momenta of only 13 tracks. The space angles of these measured tracks ranged from 15° to 45°. This certainly leaves room for one to doubt the values of momenta estimated for tracks in this angular region. If the momenta of these tracks have been overestimated, then it follows that target mass has been overestimated. In

$$\beta_c = \frac{\sum P_i \cos\theta_i}{\sum E_i} ,$$

$P_i \cos \theta_i$ becomes small at angles near 90° for a given value of P_i, while E_i is large because of overestimated momenta. The β_c becomes smaller and

$$M_{TGT} = \frac{P_o}{\beta_c} - E_o$$

is overestimated.

One concludes from the above discussion that Method I is the most correct method of evaluating effective target mass, and therefore, the values of effective target mass listed in TABLES 4 and 5 for this method will be taken as the best values.

It was proposed in Chapter I that one might test the validity of having virtual pions associated with nucleons as targets in high energy collisions by examining pion-nucleon interactions in which the nucleon recoils with low energy. The group of events with measured low-energy recoil protons are in this category. One notes, by reference to tables 4, 5, 8, and 9, that the values of target mass determined for this group differ very little from the values determined for the entire set of events. It appears then that there is little difference between the group with measured low-energy recoil protons and the entire set insofar as calculation of target mass is concerned. The value of effective target mass calculated for all events in the entire set, 156 ± 8 MeV/c^2, agrees reasonably well with values calculated by others[(5),(6),(7),(8)].

If the idea of having virtual pions as targets is to have physical meaning, then the values of effective target mass calculated for the different groups of events defined in TABLE 2 should be the same. Figure 9 is a plot of effective target mass versus the number of observed tracks per event in the different groups. It is noted that effective target mass depends on the number of tracks in the event for which it is calculated. This implies that an effective target within the proton does not exist, as such, at least for this type of collision, and that the value for effective target mass is determined by the kinematics of the events used in its calculation.

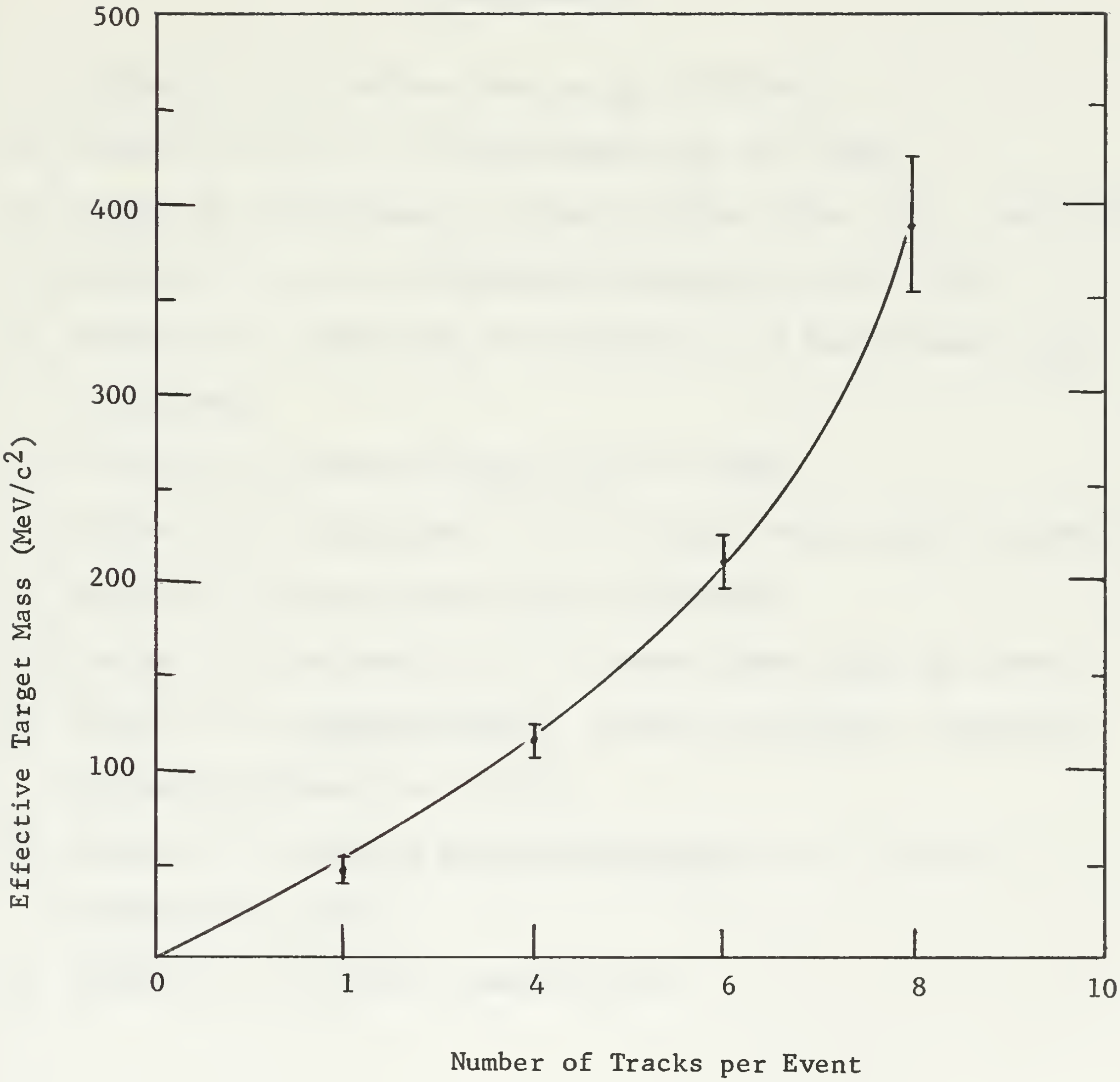

Fig. 9. Variation of Effective Target Mass for Different Groups of Events

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